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AN INDIRECT CURVE MATCHING METHOD FOR TRANSIENT MATRIX HEAT-TRANSFER TESTING IN THE LOW N_{tu}-RANGE

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Abstract—A method is described which permits the reduction of transient matrix heat-transfer test data in the low N_{tu}-range ($0{\cdot}5 < N_{tu} < 5{\cdot}0$). This method (centroid method) is based on (1) the analytic solution of the single-blow problem for arbitrary (monotone decreasing) upstream fluid temperature changes and on (2) the indirect matching of downstream fluid temperature response curves by a single-valued functional which decreases monotonically with N_{tu}, specifically by the centroid coordinate of the area under the difference of these two temperatures. Values of the centroid coordinate are presented for the case that the upstream temperature change is a step-function (sudden cooling). These data are supplemented by error amplification factors resulting from a linear error analysis. For the more general case of arbitrary upstream fluid temperature changes, two different procedures are suggested for the reduction of transient matrix heat-transfer test data: either the use of a computer which is programmed to implement the centroid method, or the use of an empirical formula which approximates the centroid coordinate.

NOMENCLATURE

h, heat-transfer coefficient [Btu/(h ft^2 degF)];
A, total heat-transfer area [ft^2];
W_s, mass of solid in core [lb];
W_f, mass of fluid [lb];
w_f, mass-flow rate of fluid [lb/h];
c_s, specific heat of solid [Btu/(lb degF)];
c_f, specific heat of fluid [Btu/(lb degF)];
x, distance from test section inlet [ft];
L, length of test section [ft];
G, temperature of the fluid (gas) [°F];
S, temperature of the solid [°F];
z, reduced length [dimensionless];
μ, "free" time [dimensionless];
N_{tu}, dimensionless parameter (number of transfer units), $N_{tu} = hA/w_f c_f$;
$(N_{tu})_{exp}$, experimentally determined value of N_{tu} for a given single-blow test;
g, normalized fluid temperature at test section inlet [dimensionless];
I, deviation from the step change [dimensionless];
I_{max}, maximal permissible deviation from the step change;
μ_{cent}, centroid coordinate.

INTRODUCTION

A METHOD to determine the heat-transfer coefficients of porous media by a suitable reduction of heat-transfer test data resulting from a single-blow test has been suggested by Saunders and Ford [1]. Their method, also known as "curve matching technique", is based on Schumann's [2] analytical solution of the single-blow problem, and is restricted to the case that the fluid temperature change at the test-section inlet is a step function. Subsequently, by introducing his maximum slope method, Locke [3] has considerably simplified the reduction of transient heat-transfer (single-blow) test data. Considerable interest in transient heat-transfer testing in the low N_{tu}-range was stimulated by the observation of Cresswick [4] and Howard [5] that the effects of longitudinal heat conduction in the test core falsifies appreciably the predictions based on Hausen's [6] model in the medium and high N_{tu}-range, but are negligible in the low N_{tu}-range. However, as the present author has demonstrated [7, 8], the maximum slope method fails for small values of N_{tu}, that is for just those N_{tu}-values where one would normally, in view of the longitudinal conduction effects, prefer to conduct the transient heat-transfer experiments. It has been suggested [7, 8] that for the low N_{tu}-range a supplementary method for the evaluation of single-blow test data is needed.

It is the purpose of this paper to present a method (centroid method) which affords the determination of the number of transfer units, N_{tu}, from single-blow test data, in the low N_{tu}-range. The characterization of fluid temperature transient response curves (functions) by their moments provides a theoretical starting point for the centroid method. To be practical, this method has to include the case of arbitrary (monotone decreasing) fluid temperature changes at the test section inlet. In developing the centroid method, two different objectives have been kept in mind: first, to create an engineering tool in the form of a computer program for efficient test-data reduction, and secondly to provide an empirical formula and a short table that can be used together in the manner of a handbook procedure (no high speed digital computer required).

REVIEW OF THE SINGLE-BLOW PROBLEM

The analysis of the extended single-blow problem is based on a modification of Hausen's mathematical model: a linear partial differential equation system of the form

$$\frac{\partial G}{\partial z} + G(z, \mu) = S(z, \mu), \tag{1}$$

$$\frac{1}{N_{tu}} \frac{\partial S}{\partial \mu} + S(z, \mu) = G(z, \mu). \tag{2}$$

This system describes the heat transfer to and from a fluid (gas) transfusing through a porous core, in terms of the fluid temperature G and the solid temperature S in the test matrix. The above system is subject to the boundary and initial conditions

$$G(z, \mu)\big|_{z=0} = g(\mu), \tag{3}$$

$$S(z, \mu)\big|_{\mu=0} = s(z). \tag{4}$$

The assumptions leading to Hausen's [6] original model are discussed in detail in such standard sources as Jakob [9] or Kays and London [10]. (One principal limitation of this model is that it neglects longitudinal heat conduction.) In equations (1–4) z is a (dimensionless) reduced length variable given by

$$z = \frac{hA}{w_f c_f} \frac{x}{L} \tag{5}$$

and μ is a (dimensionless) time variable, designated as *free time*, because it is independent of N_{tu}, the number of transfer units. The time variable μ is defined by

$$\mu = \frac{w_f c_f}{W_s c_s} t - \frac{W_f c_f}{W_s c_s} \frac{x}{L}. \tag{6}$$

It is convenient here to introduce the two constants $\alpha = w_f c_f / W_s c_s$ and $\beta = W_f c_f / W_s c_s$. The more conventional time variable *reduced time* τ is related to μ by

$$\tau = \mu N_{tu} = \left(\alpha t - \beta \frac{x}{L}\right) N_{tu} = \frac{hA}{W_s c_s} t - \frac{W_f}{w_f} \frac{x}{L}. \tag{7}$$

For most practical purposes one can set $\beta \cong 0$ and $\mu \cong \alpha t$.

As a prerequisite for any method of reducing transient matrix heat-transfer test data (single-blow test data), no matter whether one considers a direct or an indirect curve matching method, a solution of the single-blow problem is needed. For the case of an arbitrary normalized fluid temperature change, and for the normalized initial solid temperature $s(z) = 1$, an analytic solution of equations (1–4) has been derived in [11]. It is restated below:

$$G(z, \mu) = e^{-z}[g(\mu) + \int_0^\mu zN_{tu}\Xi_1 zN_{tu}(\mu - \nu)$$
$$\exp[(-N_{tu}(\mu - \nu))\, g(\nu)\, d\nu] + 1 - e^{-z}$$
$$\times [1 + \int_0^\mu z\, N_{tu}\, \Xi_1(zN_{tu}\, \nu) \exp(-N_{tu}\nu)\, d\nu]. \tag{8}$$

In both of the above integrands, Ξ_1 (.) denotes an entire function defined by

$$\Xi_1(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!(n+1)!}. \tag{9}$$

The (numerical) computation of $G(z, \mu)$ as represented by equation (8) poses no problem; $G(z, \mu)$ can be evaluated for as many values of z as desirable. However, in most curve matching (data reduction) problems, only the fluid temperature response at the test-section exit is of particular interest. In this context it is interesting to note that

$$z|_{x=L} = N_{tu}. \tag{10}$$

The curve matching problem consists in finding a value for the *parameter* N_{tu} such that the experimentally determined temperature response $G_{exp}(\mu)$ "matches" in some sense one particular element $G(N_{tu}, \mu) = G(z, \mu)|_{x=L}$ of the one-parameter family of response functions, $G(N_{tu}, \mu$.

Ideally, that is for the case of complete agreement between theory and experiment, one has for the difference between theoretical and experimental response functions

$$\Delta G \equiv G(N_{tu}, \mu) - G_{exp}(\mu) = 0, \tag{11}$$

an equation which implies, for all non-negative integers l, that the moments of the temperature difference ΔG are zero,

$$\int_0^\infty \mu^l\, \Delta G\, d\mu = 0, \tag{12}$$

or, equivalently, that

$$\int_0^\infty \mu^l\, G(N_{tu}, \mu)\, d\mu = \int_0^\infty \mu^l\, G_{exp}(\mu)\, d\mu,$$
$$(l = 0, 1, 2, \ldots). \tag{13}$$

For this reason one is interested in the moments of lth order of the theoretical fluid temperature response function $G(z, \mu)$, which are defined by

$$M_l(z) = \int_0^\infty \mu^l\, G(z, \mu)\, d\mu, (l = 0, 1, 2, \ldots). \tag{14}$$

The above integrations are very difficult to perform in a straightforward manner. In Appendix 1 it will be shown how these moments can be evaluated by using Laplace transform theory; there results

$$M_l(z) = \frac{l!}{N_{tu}^{l+1}} \sum_{i=0}^{l} \binom{l}{i} \frac{z^{i+1}}{(i+1)!}$$
$$+ \sum_{k=0}^{l} \binom{l}{k} \left[\delta_{0,k} + \frac{(1 - \delta_{k,0})\, k!}{N_{tu}^k} \right.$$
$$\left. \times \sum_{i=0}^{k-1} \binom{k-1}{i} \frac{z^{i+1}}{(i+1)!} \right] \int_0^\infty \mu^{l-k}\, g(\mu)\, d\mu. \tag{15}$$

For *actual* experiments one finds that

$$\Delta G = G(N_{tu}, \mu) - G_{exp}(\mu) \neq 0 \tag{16}$$

and, consequently, one has to relax the requirement expressed by equation (13). In practical applications one is interested only in moments of low order, evaluated at the test section exit, that is for $z = N_{tu}$. For the first three moments one has then, according to equation (15),

$$M_0(N_{tu}) = 1 + \int_0^\infty g(\mu)\, d\mu, \tag{17}$$

$$M_1(N_{tu}) = \frac{1}{2} + \frac{1}{N_{tu}} + \int_0^\infty \mu g(\mu)\, d\mu + \int_0^\infty g(\mu)\, d\mu, \tag{18}$$

$$M_2(N_{tu}) = \frac{2}{N_{tu}^2} + \frac{2}{N_{tu}} + \frac{1}{3} + \int_0^\infty \mu^2 g(\mu)\,\mathrm{d}\mu + 2\int_0^\infty \mu\, g(\mu)\,\mathrm{d}\mu + \left(\frac{2}{N_{tu}} + 1\right)\int_0^\infty g(\mu)\,\mathrm{d}\mu. \quad (19)$$

One recognizes readily that the l-th order moment of the downstream fluid temperature depends on the moments up to l-th order of the upstream fluid temperature.

SELECTION OF A CURVE MATCHING METHOD

The problem of curve matching can be approached via direct and indirect methods. *Direct curve matching* is conceptually simple, and reliable as well: by defining a suitable "distance" between the experimental and one theoretical response, for example

$$\mathrm{d}(N_{tu}) = \mathrm{d}(G_{\exp}(\mu), G(N_{tu}, \mu)) = \left[\int_0^\infty (G_{\exp}(\mu) - G(N_{tu}, \mu))^2\,\mathrm{d}\mu\right]^{\frac{1}{2}}, \quad (20)$$

the curve matching problem can be reduced to the extremal problem of finding a theoretical response $G(N_{tu}, \mu)$ such that $\mathrm{d}(N_{tu})$ be a minimum. Only for the ideal (unrealistic) case that the experimental response $G_{\exp}(\mu)$ coincides with the corresponding (matching) $G(N_{tu}, \mu)$ would the "distance" $\mathrm{d}(N_{tu})$ be zero. In praxi, $G_{\exp}(\mu)$ deviates somewhat from the matching $G(N_{tu}, \mu)$, either because of instrumentation errors or because of the crudeness of Hausen's mathematical model. Direct curve matching is not always desirable since the computational effort is considerable. At any rate, it may be recalled here that the magnitude of the computational effort has motivated Locke [3] to propose his maximum slope method.

In view of the above assessment, it seems natural to look for an alternate approach toward curve matching. *Indirect curve matching* is based on the following simple idea. If the experimental data representing $G_{\exp}(\mu)$ can be reduced to a single (real) number, say $\varphi_{\exp}$, and if the data representing the whole one-parameter family $G(N_{tu}, \mu)$ of theoretical responses can, in exactly the same manner, be reduced to a (real) single-valued strictly monotone function, say $\varphi(N_{tu})$, of the parameter N_{tu} alone, then it should in principle, be possible to find that value of N_{tu} for which the equation $\varphi_{\exp} = \varphi(N_{tu})$ is satisfied. (This idea is suggested by a consideration of the maximum slope method.)

The reduction of data, no matter whether experimental or theoretical, is accomplished by means of a *mapping* which maps functions into (real) numbers. (Such a mapping is termed *functional*.) Our task is then to find a suitable real single-valued continuous functional Φ: $G(N_{tu}, \mu \to \varphi(N_{tu})$, defined on the one-parameter family of theoretical response functions $G(N_{tu}, \mu)$, such that the values of this functional, $\varphi(N_{tu}) = \Phi G(N_{tu}, \mu)$ depend strictly monotonically on the parameter, N_{tu}. The same functional Φ has to include the mapping $G_{\exp}(\mu) \to \varphi_{\exp}$ for the experimental temperature response, where $\varphi_{\exp} = \Phi G_{\exp}(\mu)$. After having somehow found the function φ, one can compute the unknown parameter N_{tu} by first evaluating $\varphi_{\exp}$ and then evaluating the function inverse to φ, namely φ^{-1}, for $\varphi_{\exp}$:

$$(N_{tu})_{\exp} = \varphi^{-1}(\varphi_{\exp}). \quad (21)$$

In general, the inverse function φ^{-1} is not given explicitly. In this case, the determination of the parameter N_{tu} reduces to a problem of interpolation.

To account for the effect of an "arbitrary" fluid temperature at the test-section inlet, $g(\mu)$, it is desirable to generalize somewhat the approach outlined above by choosing a mapping which depends not only on $G(N_{tu}, \mu)$ but on $g(\mu)$ as well. Thus we are led to consider the functional Ψ: $G(N_{tu}, \mu), g(\mu) \to \psi(N_{tu})$. The rest of the argument remains intact. No general rule is known for designing suitable functionals for indirect curve matching.

No attempt will be made in this paper to derive a suitable curve matching functional from general principles; rather, a heuristic approach, based on practical considerations, will be preferred. It is, nevertheless, important to review the conditions to be imposed on such a functional. In the following let $\psi(N_{tu})$ denote the value of the (general) curve matching functional Ψ,

$$\psi(N_{tu}) = \Psi(G(N_{tu}, \mu), g(\mu)). \tag{22}$$

For Ψ to be suitable for indirect curve matching, the following is necessary:

(i) Ψ must be real, single-valued, continuous with respect to both $G(N_{tu}, \mu)$ and $g(\mu)$, and strictly monotone (either increasing or decreasing) with the parameter N_{tu};

(ii) $\psi(N_{tu})$ must be strictly monotone increasing with $I(g) = \int_0^\infty g(\mu)\,d\mu$, the *deviation from the step*;

(iii) $\left|\frac{d}{dN_{tu}}\psi(N_{tu})\right| \geqslant k$, where k is some measure for the maximal permissible amplification of errors;

(iv) for any N_{tu} and any given deviation from the step, $\psi(N_{tu})$ should be insensitive with respect to local variations of $g(\mu)$;

(v) the evaluation of $\psi(N_{tu})$ must be simple and straightforward.

These five conditions are dictated by practical considerations. If (i) is not satisfied, any number of values for $(N_{tu})_{exp}$ may result, or $(N_{tu})_{exp}$ may be indeterminate. If (ii) is not met, then the concept "deviation from the step" becomes rather useless, and no upper or lower error bounds can be established. If (iii) is violated, intolerable errors in N_{tu} may result. Condition (iv) assures that the conclusions drawn for one particular $g(\mu)$ can be extended meaningfully to a slightly varied change. Finally, if (v) is not satisfied, then the intended indirect curve matching method offers no advantage over a direct curve matching method.

Apart from its failure in the low N_{tu}-range, the (extended) maximum slope method (see [8]) can be criticized on other grounds. First, we note its great dependence on high quality first derivatives of $G_{exp}(\mu)$ and $g(\mu)$. (As far as the free time at maximum slope is concerned, even high quality second derivatives are required.) It is an experimental fact, that most graphically recorded temperature responses exhibit high frequency oscillations ("wiggles") due to signal amplifier noise. Whereas in the visual–manual application of the (extended) maximum slope method it is possible to smooth the recorded temperature curves before (or while) determining the maximum slope, an automated heat-transfer testing procedure, which is being aimed at here, would require a well-defined and reliable smoothing routine which is not readily available and which would only unnecessarily complicate matters. In the second place, one does well to recognize that the (extended) maximum slope method provides no means of establishing close error bounds (see [8]).

The above arguments suggest looking for curve matching functionals of integral form. On this basis, it is natural to consider low order moments of the fluid temperature response, or modifications thereof, as curve matching functionals. Equations (17) and (18) can be rewritten, for all $N_{tu} > 0$, as

$$\int_0^\infty [G(N_{tu}, \mu) - g(\mu)]\,d\mu = 1, \tag{23}$$

$$\int_0^\infty \mu[G(N_{tu}, \mu) - g(\mu)]\,d\mu = \frac{1}{2} + \frac{1}{N_{tu}} + \int_0^\infty g(\mu)\,d\mu. \tag{24}$$

Whereas the zero order moment does not qualify as a curve matching functional, because it is independent of N_{tu}, the first order moment of the difference between upstream and downstream fluid temperatures is in many respects a suitable curve matching functional [12].

The first moment of the difference $G_{exp}(\mu) - g(\mu)$, where $G_{exp}(\mu)$ stands for the normalized experimental fluid temperature response at the test-section exit, is known for each experiment.

The matching of the experimental response curve with one of the theoretical response curves is accomplished by requiring that the equation

$$\int_0^\infty \mu[G_{\exp}(\mu) - g(\mu)]\,d\mu = \frac{1}{2} + \frac{1}{(N_{tu})_{\exp}} + \int_0^\infty g(\mu)\,d\mu \qquad (25)$$

holds, where $(N_{tu})_{\exp}$ denotes that N_{tu}-value for which curve matching is achieved. One finds at once that

$$(N_{tu})_{\exp} = \left[\int_0^\infty \mu[G_{\exp}(\mu) - g(\mu)]\,d\mu - \frac{1}{2} - \int_0^\infty g(\mu)\,d\mu\right]^{-1}. \qquad (26)$$

Equation (26) is an explicit formula for the reduction of single-blow test-data. Its value is limited because it is practically impossible to integrate up to the free time $\mu = \infty$. The term on the left side of equation (24) has a geometrical significance: it is the (free time) *centroid coordinate* of the area between $G(N_{tu}, \mu) - g(\mu)$ and the μ-axis.

The definition of the following two quantities is useful:

$$I(g) \equiv \int_0^\infty g(\mu)\,d\mu, \qquad (27)$$

which we term *deviation from the step*, and

$$M(g) \equiv \int_0^\infty \mu[G(N_{tu}, \mu) - g(\mu)]\,d\mu, \qquad (28)$$

the first order moment of the temperature difference. With this notation, equation (24) can be restated as

$$M(g) = \frac{1}{2} + \frac{1}{N_{tu}} + I(g), \text{ for all } N_{tu} > 0. \qquad (29)$$

It follows at once, for arbitrary upstream temperature changes $g_1(\mu)$ and $g_2(\mu)$, that

$$M(g_2) - M(g_1) = I(g_2) - I(g_1), \qquad (30)$$

that is the difference between first moments can be expressed in terms of the difference of the deviations. Moreover, under the more restrictive assumption that both $g_1(\mu)$ and $g_2(\mu)$ are non-negative monotone decreasing, there follows that $g_2(\mu) \geqslant g_1(\mu)$ for all $\mu \geqslant 0$ implies both $I(g_2) \geqslant I(g_1)$ and $M(g_2) \geqslant M(g_1)$. Thus it is a simple matter to establish error bounds on first moments.

It can be verified that the first moment of the temperature difference, $M(g)$, satisfies conditions (i), (ii), and (iv). Condition (iii) indicates that $M(g)$ as a curve matching functional is useful only in the low N_{tu}-range. Finally, examining the usefulness of $M(g)$ as a curve matching functional with respect to condition (v) shows another shortcoming: it is practically impossible to integrate up to free time $\mu = \infty$. Therefore, a modification of the first moment approach is necessary.

DESCRIPTION OF THE CENTROID METHOD

The centroid method results from a practical adaptation of the first moment approach to indirect curve matching. Instead of extending the integration up to free time $\mu = \infty$, it is practical to "clip-off the tail", that is, to evaluate the first moment integral within *finite* limits of integration. This can, in principle, be done in two different ways: either as a *fixed clip-off*, or as a *response dependent clip-off*. The first of these two possibilities is conceptually simpler, but gives rise to greater amplification of relative errors, and is, for the case of arbitrary upstream fluid temperature changes, not very suitable as an off-computer method. The fixed clip-off approach will be briefly discussed in Appendix 2. In the following, we will be concerned with a response dependent clip-off. The deviations $I(g)$ from the step change must be bounded. Accordingly, let $I_{\max}$ denote the maximal permissible deviation from the step change.

It is necessary to restrict the class of "arbitrary" upstream fluid temperature changes to those functions $g(\mu)$ which

(i) are non-negative monotone decreasing,
(ii) have initial value $g(0) = g(\mu)|_{\mu=0} = 1$, and
(iii) assume value zero for all free times which exceed the maximal permissible deviation from the step: $g(\mu) = 0$ for all $\mu \geqslant I_{\max}$.

Condition (i) hardly imposes any serious experimental limitations. The normalized condition (ii) is imposed to simplify data reduction and does not affect the performance of the single-blow experiment. Only condition (iii) puts a slight burden on the experimentalist by demanding of him to provide temperature control equipment which is flexible enough to keep the upstream fluid temperature constant after a certain finite free time $\mu = I_{\max}$. (Owing to the use of *normalized* upstream temperature changes $g(\mu)$ one has, for functions which satisfy condition (iii), that $I = \int_0^{I_{\max}} g(\mu)\,d\mu \leqslant I_{\max}$, an inequality which helps explain why the domain of integration can be restricted by $I_{\max}$.)

In the centroid method the integration is performed up to the so-called *one-tenth value* of the free time. Among all $\mu \geqslant I_{\max}$, define $\mu_{1/10}$ to be that number for which $G(N_{tu}, \mu) - g(\mu) = 1/10$ (see Fig. 1). The actual computation of $\mu_{1/10}$ is a simple problem of interpolation once $G(N_{tu}, \mu)$ is known for a discrete set of μ's.

The area under the normalized temperature difference $G(N_{tu}, \mu) - g(\mu)$, bounded at the right by $\mu = \mu_{1/10}$, has a centroid; the μ-coordinate of the centroid is given by

$$\mu_{\text{cent}}(N_{tu}; g) = \frac{\int_0^{\mu_{1/10}} \mu[G(N_{tu}, \mu) - g(\mu)]\,d\mu}{\int_0^{\mu_{1/10}} [G(N_{tu}, \mu) - g(\mu)]\,d\mu}. \quad (31)$$

The centroid coordinate thus defined is a function of the parameter N_{tu} and can be computed for as many values of N_{tu} as necessary; it is a functional of both $g(\mu)$ and $G(N_{tu}, \mu)$.

For the experimental temperature response

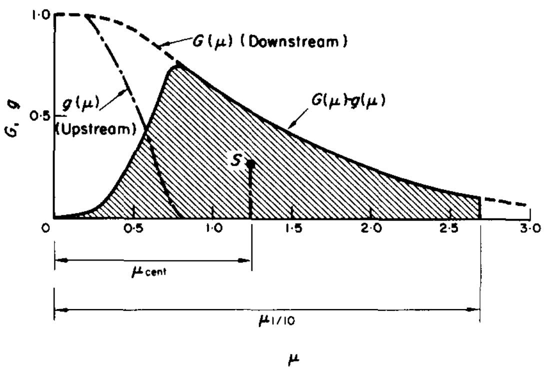

FIG. 1. Centroid and centroid coordinate ($N_{tu} = 2{\cdot}5$).

$G_{\exp}(\mu)$ one can proceed in a similar manner. Define $(\mu_{1/10})_{\exp}$ to be that number μ for which $G_{\exp}(\mu) - g(\mu) = 1/10$. Define and compute the experimental centroid coordinate,

$$[\mu_{\text{cent}}(g)]_{\exp} = \frac{\int_0^{(\mu_{1/10})_{\exp}} \mu[G_{\exp}(\mu) - g(\mu)]\,d\mu}{\int_0^{(\mu_{1/10})_{\exp}} [G_{\exp}(\mu) - g(\mu)]\,d\mu}. \quad (32)$$

Then the experimental N_{tu}-value, to be denoted by $(N_{tu})_{\exp}$, is determined as that number N_{tu} which satisfied the equation

$$\mu_{\text{cent}}(N_{tu}; g) = [\mu_{\text{cent}}(g)]_{\exp}. \quad (33)$$

The actual determination of $(N_{tu})_{\exp}$ is again a simple problem of interpolation. A computing program [13] based on the procedure described above has been written, tested, and successfully used in the low N_{tu}-range. However, some caution is necessary: for *extremely small* N_{tu}-values it can happen that $\mu_{1/10}$ does not exist, but this is hardly ever of practical consequence. [The reason for the possible non-existence of $\mu_{1/10}$ is the following. Inspection of equation (7) shows that as $N_{tu} \to 0$ so $G(N_{tu}, \mu) - g(\mu) \to 0$. Thus $G(N_{tu}, \mu) - g(\mu)$ does not necessarily assume the value 1/10 for all small N_{tu}-values.]

Of particular interest is, of course, the limiting case of instantaneous (step change) cooling. The fluid temperature change at the test section inlet is then described by

$$g(\mu) = \begin{cases} 1 \ (\mu \leqslant 0), \\ 0 \ (\mu > 0), \end{cases} \tag{34}$$

and one has

$$\mu_{\text{cent}}(N_{tu}; 0) = \frac{\int_0^{\mu_{1/10}} \mu G(N_{tu}, \mu)\, \mathrm{d}\mu}{\int_0^{\mu_{1/10}} G(N_{tu}, \mu)\, \mathrm{d}\mu} \tag{35}$$

for the resulting centroid coordinate. Values of $\mu_{\text{cent}}(N_{tu}; 0)$ are presented in Table 1; for a graphical representation see Fig. 2.

Table 1. Centroid coordinates

N_{tu}	$\mu_{\text{cent}}(N_{tu}; 0)$
0·5	1·365
1·0	1·036
1·5	0·884
2·0	0·799
2·5	0·744
3·0	0·707
3·5	0·679
4·0	0·658
4·5	0·641
5·0	0·628

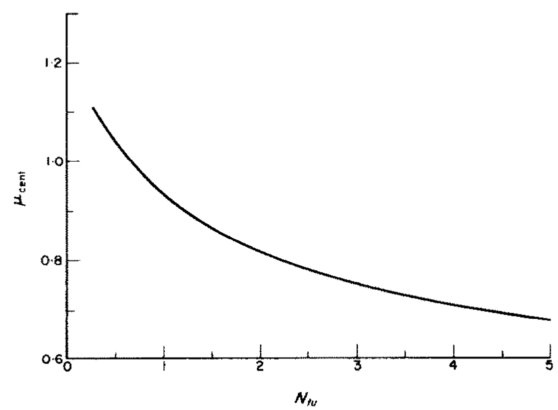

FIG. 2. Centroid coordinates: μ_{cent} vs. N_{tu}.

Table 1 presents the centroid coordinates $\mu_{\text{cent}}(N_{tu}; 0)$ due to a step-change of the inlet fluid temperature as a function of N_{tu}.

LINEAR ERROR ANALYSIS

No data reduction method should be considered complete without a companion error analysis. A preliminary estimate of the amplification of relative errors arising in the centroid method can be obtained by studying equations (23) and (24). Thus we consider the (unclipped) centroid coordinate

$$\mu_{\text{cent}}(N_{tu}) = \frac{\int_0^{\infty} \mu[G(N_{tu}, \mu) - g(\mu)]\, \mathrm{d}\mu}{\int_0^{\infty} [G(N_{tu}, \mu) - g(\mu)]\, \mathrm{d}\mu} = \frac{1}{2} + \frac{1}{N_{tu}} + I(g). \tag{36}$$

A linear (or first order) error analysis is based on the approximation

$$\Delta N_{tu} \cong \frac{\mathrm{d}N_{tu}}{\mathrm{d}\mu_{\text{cent}}} \Delta\mu_{\text{cent}}; \tag{37}$$

introducing the relative error amplification factor

$$K = \frac{\mu_{\text{cent}}}{N_{tu}} \frac{\mathrm{d}N_{tu}}{\mathrm{d}\mu_{\text{cent}}}, \tag{38}$$

one finds for the relative errors the relation

$$\frac{\Delta N_{tu}}{N_{tu}} \cong K \frac{\Delta\mu_{\text{cent}}}{\mu_{\text{cent}}}. \tag{39}$$

Accordingly, K is that factor by which the relative errors in the centroid coordinate have to be multiplied to obtain, to first order approximation, the relative errors in the N_{tu}-value. There results

$$K = -\frac{\mu_{\text{cent}}}{\mu_{\text{cent}} - \frac{1}{2} - I(g)} = -\{1 + [\tfrac{1}{2} + I(g)]\, N_{tu}\}. \tag{40}$$

The corresponding error analysis for the centroid method (response dependent clip-off at $\mu_{1/10}$) is not as simple and must be carried out numerically; the results of such an analysis for the case of a step change are shown in Figs. 3 and 4. Inspection of Fig. 4 shows that best data

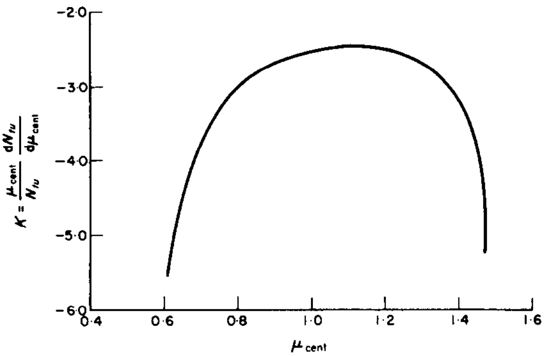

FIG. 3. Amplification of relative errors: K vs. μ_{cent}.

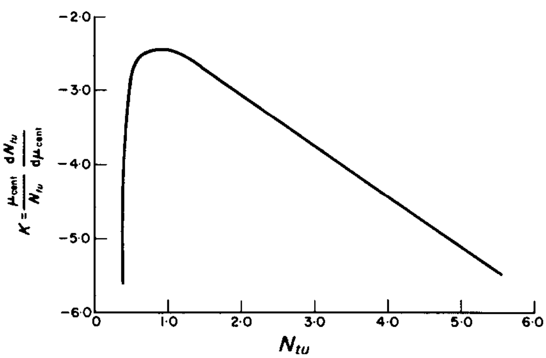

FIG. 4. Amplification of relative errors: K vs. N_{tu}.

reduction by the centroid method can be expected in the N_{tu}-range $0{\cdot}5 < N_{tu} < 5{\cdot}0$, corresponding to an amplification of relative errors not much worse than $K = -5{\cdot}0$. Thus, for instance, if μ_{cent} is measured too high by 2 per cent and if $K = -5{\cdot}0$, then N_{tu}, as obtained by the centroid method, would be too low by about 10 per cent.

ESTIMATES, EMPIRICAL FORMULA AND ACCURACY

"Clipping off the tail" of the temperature response $G(N_{tu}, \mu)$ has the undesired effect that the simple formula (24) of the first moment approach is no longer valid. Nevertheless we wish to demonstrate that the deviation from the step $I(g)$ is a meaningful and useful quantity in the centroid method and that equation (24) can be replaced by an empirical formula.

Whenever the functional "centroid coordinate", as defined by equation (31) exists for two different non-negative monotone decreasing upstream temperature changes $g_1(\mu)$ and $g_2(\mu)$ which are such that

$$g_2(\mu) \geqslant g_1(\mu) \quad (\text{all } \mu \geqslant 0), \tag{41}$$

then it can be concluded that

$$\mu_{cent}(N_{tu}; g_2) \geqslant \mu_{cent}(N_{tu}; g_1). \tag{42}$$

Thus, as illustrated in Fig. 5, the value of the centroid coordinate due to $g_2(\mu)$ exceeds that which is due to $g_1(\mu)$. This important boundedness property makes it possible to derive *estimates* for $\mu_{cent}(N_{tu}; g)$ by evaluating upper and lower bounds on the latter quantity. At worst, such bounds may be evaluated for *delayed step changes* (see [12]) which bound $g(\mu)$ from above and below. Actually, a lot more can be done by relying on the *empirical formula*

$$\mu_{cent}(N_{tu}; g) \cong \mu_{cent}(N_{tu}; 0) + 0{\cdot}99\, I(g) \pm 0{\cdot}01 \tag{43}$$

which has been obtained by systematically applying the centroid method to many different upstream fluid temperature changes. Formula (43) can be expected to hold for all $I(g) \leqslant 0{\cdot}5$ in the N_{tu}-range $0{\cdot}5 < N_{tu} < 5{\cdot}0$ and represents, together with Table 1, the replacement of formula (24). Although equation (43) is not an exact result, quite accurate values of N_{tu} can be obtained by applying it in indirect curve matching. It may be noted that, in the sense of this "approximation", the centroid coordinate due to $g(\mu)$ is determined by the deviation from the step-change alone, and is thus more or less independent of local variations from $g(\mu)$.

The "shape" (or "profile") of the transient temperature response due to some upstream temperature change is known to be independent of the specific instant when the single-blow test is initiated (see [12]). That means that the transient response due to a delayed upstream temperature change is equal to the delayed response due to the undelayed upstream temperature change.

In view of this fact, formula (43) can be made somewhat plausible by considering a delayed

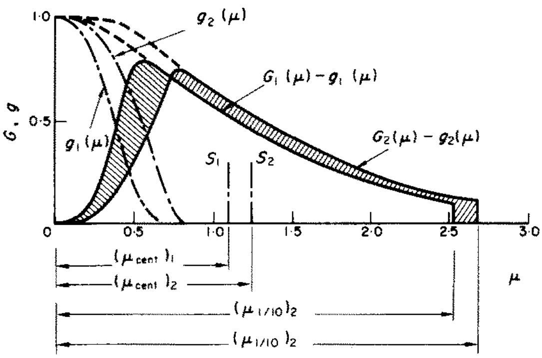

FIG. 5. Centroid for two different upstream fluid temperatures.

step-change. Let $g(\mu)$ be a step-change delayed by a (free) time shift $\Delta\mu$ so that

$$I(g) = \int_0^{\mu_{1/10}} g(\mu)\,d\mu = \Delta\mu.$$

In terms of centroid coordinates one has then

$$\mu_{cent}(N_{tu}; g) = \mu_{cent}(N_{tu}; 0) + \Delta\mu, \quad (44)$$

in agreement with formula (24).

For the case of an arbitrary upstream fluid temperature change, an error analysis based on the empirical formula (43) can be carried out. However, for most practical purposes and in particular for small deviations from the step, it suffices to consider the amplification of relative errors as illustrated in Fig. 4.

The accuracy of a curve matching method can be determined independent of single-blow test by *simulating* the experimental downstream fluid temperatures by their theoretical counterparts, and by accepting the theoretical instead of the experimental temperatures in the computational procedure leading to $(N_{tu})_{exp}$. The accuracy of the centroid method has been established in this manner, to at least three significant figures in the computer program version and to at least two significant figures in the "handbook" version of the centroid method.

CONCLUSION

In using Hausen's analysis to reduce transient matrix heat-transfer test data one has faced the following dilemma. Hausen's mathematical model is, in the presence of longitudinal thermal conduction, acceptable only for small N_{tu}-values, but for small N_{tu}-values the conventional method for test-data reduction, the maximum slope method, fails. To reduce single-blow test-data according to Hausen's model a new method, the centroid method, may be used. This method is limited to N_{tu}-values in the range $0{\cdot}5 < N_{tu} < 5{\cdot}0$.

The results of this paper should be useful in three respects.

1. If large test series are to be performed, an automated test procedure, based on a well-defined curve-matching method is needed. To automate the procedure, the conventional test equipment can be complemented with a paper tape puncher which is to be connected to the temperature recording instruments. The paper tape puncher provides the input for the digital computer which is programmed to solve the curve-matching problem by means of the centroid method.
2. For the occasional reducing of single-blow test data, the use of an empirical formula together with a short table is suggested. (No digital computer required.)
3. Analytical expressions have been derived for the moments of all orders of the transient fluid temperature response function. These moments are of possible use in future treatments of both the curve matching problem and a detailed appraisal of Hausen's mathematical model.

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APPENDIX 1

Moments of the Fluid Temperature Response

In this Appendix we show how to evaluate the moments of the fluid temperature response $G(z, \mu)$, as defined by equation (14) of the main text. It is convenient to decompose the response function $G(z, \mu)$ given by equation (8), into two parts,

$$G(z, \mu) = G_1(z, \mu) + G_g(z, \mu), \tag{45}$$

and to evaluate the moments of the two parts separately. We begin with the moments of

$$G_1(z, \mu) = 1 - e^{-z}[1 + \int_0^\mu zN_{tu} \times \Xi_1(zN_{tu}\nu) \exp(-N_{tu}\nu)\, d\nu]. \tag{46}$$

The Laplace transform of this function, with s denoting the Laplace transform parameter, is given by

$$\bar{G}_1(z, s) = \mathscr{L}\{G_1(z, \mu); s\} = \int_0^\infty G_1(z, \mu)\, e^{-s\mu}\, d\mu = \frac{1}{s}\left[1 - \exp\left(-\frac{zs}{s + N_{tu}}\right)\right] = \sum_{i=0}^\infty \frac{(-1)^i s^i}{(i+1)!}\left(\frac{z}{s + N_{tu}}\right)^{i+1}. \tag{47}$$

Differentiation of equation (47) with respect to s yields

$$\frac{d\bar{G}_1(z, s)}{ds} = -\int_0^\infty \mu G_1(z, \mu)\, e^{-s\mu}\, d\mu = \mathscr{L}\{-\mu G_1(z, \mu); s\}. \tag{48}$$

Repeating this process l times, and taking then the limit as $s \to 0+$, one finds

$$\lim_{s\to 0+}\left[(-1)^l \frac{d^l\bar{G}_1(z, s)}{ds^l}\right] = \lim_{s\to 0+}\int_0^\infty \mu^l G_1(z, \mu)\, e^{-s\mu}\, d\mu = \int_0^\infty \mu^l G_1(z, \mu)\, d\mu \equiv M_{1,l}(z), \tag{49}$$

the moment of lth order of $G_1(z, \mu)$. Substitution of the series expansion

$$s^i = [(s + N_{tu}) - N_{tu}]^i = \sum_{j=0}^i (-1)^j \times \binom{i}{j} (s + N_{tu})^{i-j} N_{tu}^j \tag{50}$$

in equation (47) gives the double series

$$\bar{G}_1(z, s) = \sum_{i=0}^{\infty} \frac{(-1)^i z^{i+1}}{(i+1)!} \sum_{j=0}^{i} \frac{(-1)^j}{N_{tu}} \times \binom{i}{j} \left(\frac{N_{tu}}{s + N_{tu}}\right)^{j+1}, \quad (51)$$

and by differentiating equation (51) l times one obtains

$$(-1)^l \frac{d^l \bar{G}(z, s)}{ds} = l! \sum_{i=0}^{\infty} \frac{(-1)^i z^{i+1}}{(i+1)!} \times \sum_{j=0}^{i} \frac{(-1)^j}{N_{tu}} \binom{i}{j} \binom{j+l}{j} \frac{N_{tu}^{j+1}}{(s + N_{tu})^{l+j+1}}. \quad (52)$$

One finds thus the limit

$$\lim_{s \to 0+} \left[(-1)^l \frac{d^l G_1(z, s)}{ds^l}\right] = l! \times \sum_{i=0}^{\infty} \frac{(-1)^i z^{i+1}}{(i+1)!\, N_{tu}^{l+1}} \sum_{j=0}^{i} (-1)^j \binom{i}{j} \binom{j+l}{j}. \quad (53)$$

In order to further reduce the right side of equation (53) we note the algebraic identity

$$\sum_{j=0}^{i} (-1)^j \binom{i}{j} \binom{j+l}{j} = (-1)^i \binom{l}{i} \sum_{j=0}^{i} \delta_{i,j}, \quad (54)$$

where $\delta_{i,j}$ is the Kronecker symbol. Substituting equation (54) in equation (53) we arrive at

$$M_{1,l}(z) = \int_0^{\infty} \mu^l G_1(z, \mu)\, d\mu = \frac{l!}{N_{tu}^{l+1}} \sum_{i=0}^{l} \binom{l}{i} \frac{z^{i+1}}{(i+1)!}, \quad (55)$$

the desired moments for the first part of the response function.

We turn now to the evaluation of the moments of the second part of the response function,

$$G_g(z, \mu) = e^{-z}\{g(\mu) + \int_0^{\mu} z N_{tu} \Xi_1 (z N_{tu}(\mu - \nu)) \times \exp[-N_{tu}(\mu - \nu)]\, g(\nu)\, d\nu\}. \quad (56)$$

Here we find for the Laplace transform of $G_g(z, \mu)$ the product

$$\bar{G}_g(z, s) = \mathscr{L}(G_g(z, \mu); s) = \bar{K}(z, s)\, \bar{g}(s), \quad (57)$$

where

$$\bar{K}(z, s) = \mathscr{L}(K(z, \mu); s) = \mathscr{L}\{e^{-z}[\delta(\mu) + z N_{tu} \Xi_1(z N_{tu} \mu) \exp(-N_{tu}\mu)]; s\} = \exp\left(-\frac{zs}{s + N_{tu}}\right), \quad (58)$$

and

$$\bar{g}(s) = \mathscr{L}(g(\mu); s). \quad (59)$$

For the lth order derivative of the Laplace transform of $G_g(z, \mu)$ one has then

$$(-1)^l \frac{d^l G_g(z, s)}{ds^l} = \sum_{k=0}^{l} \binom{l}{k} (-1)^k \frac{d^k \bar{K}(z, s)}{ds^k} \times (-1)^{l-k} \frac{d^{l-k} \bar{g}(s)}{ds^{l-k}}. \quad (60)$$

We proceed to rewrite $\bar{K}(z, s)$ as follows:

$$\bar{K}(z, s) = \exp\left[-z\left(1 - \frac{N_{tu}}{s + N_{tu}}\right)\right] = \sum_{i=0}^{\infty} \frac{(-z)^i}{i!} \sum_{j=0}^{i} (-1)^j \binom{i}{j} \left(\frac{N_{tu}}{s + N_{tu}}\right)^j$$

$$= 1 + \sum_{i=0}^{\infty} \frac{(-z)^{i+1}}{(i+1)!} \left[1 + \sum_{j=0}^{i} (-1)^{j+1} \binom{i+1}{j+1} \left(\frac{N_{tu}}{s + N_{tu}}\right)^{j+1}\right]$$

$$= e^{-z} + \sum_{i=0}^{\infty} \frac{(-z)^{i+1}}{(i+1)!} \sum_{j=0}^{i} (-1)^{j+1} \binom{i+1}{j+1} \left(\frac{N_{tu}}{s + N_{tu}}\right)^{j+1}. \quad (61)$$

Similarly as before, one finds

$$(-1)^k \frac{\mathrm{d}^k \bar{K}(z,s)}{\mathrm{d}s^k} = \mathrm{e}^{-z}\,\delta_{0,k} + k! \sum_{i=0}^{\infty} \frac{(-1)^i z^{i+1}}{(i+1)!} \sum_{j=0}^{i} (-1)^j \binom{i+1}{j+1}\binom{j+k}{j} \frac{N_{tu}^{j+1}}{(s+N_{tu})^{k+j+1}}; \quad (62)$$

the limit of equation (62), as $s \to 0+$, becomes

$$N_k(z) \equiv \lim_{s\to 0+} \left[(-1)^k \frac{\mathrm{d}^k \bar{K}(z,s)}{\mathrm{d}s^k}\right]$$

$$= \mathrm{e}^{-z}\,\delta_{0,k} + \frac{k!}{N_{tu}^k} \sum_{i=0}^{\infty} \frac{(-1)^i z^{i+1}}{(i+1)!} \sum_{j=0}^{i} (-1)^j \binom{i+1}{j+1}\binom{j+k}{j}. \quad (63)$$

The last double series can be simplified by substituting the algebraic identity

$$\sum_{j=0}^{i} (-1)^j \binom{i+1}{j+1}\binom{j+k}{j} = -\delta_{0,k} + (1-\delta_{0,k})(-1)^i \binom{k-1}{i} \sum_{j=0}^{k-1} \delta_{i,j} \quad (64)$$

in equation (63). There follows

$$N_k(z) = \delta_{0,k} + \frac{(1-\delta_{0,k})\,k!}{N_{tu}^k} \sum_{i=0}^{k} \binom{k-1}{i} \frac{z^{i+1}}{(i+1)!}, \quad (65)$$

and we obtain for the moments of the second part of $G(z, \mu)$ the expression

$$M_{g,l}(z) = \sum_{k=0}^{l} \binom{l}{k} N_k(z)\, m_{l-k}, \quad (66)$$

where

$$m_{l-k} = \int_0^{\infty} \mu^{l-k} g(\mu)\,\mathrm{d}\mu = \lim_{s\to 0+} \left[(-1)^{l-k} \frac{\mathrm{d}^{l-k}\bar{g}(s)}{\mathrm{d}s^{l-k}}\right] \quad (67)$$

is the $(l-k)$-th order moment of the upstream fluid temperature change $g(\mu)$.

APPENDIX 2

Fixed Clip-off

The purpose of this Appendix is to outline a somewhat different adaptation of the first moment approach to curve matching. A simple procedure consists in "clipping off" the tail of the transient response at a *fixed* value of the free time μ, say at $\mu = 2{\cdot}5$. As a curve matching functional one might then use the centroid coordinate

$$\mu_{\text{cent}}(N_{tu}; g) = \frac{\int_0^{2\cdot 5} \mu[G(N_{tu}, \mu) - g(\mu)]\,\mathrm{d}\mu}{\int_0^{2\cdot 5} [G(N_{tu}, \mu) - g(\mu)]\,\mathrm{d}\mu}, \quad (68)$$

which is, for the case of sudden cooling $[g(\mu) \equiv 0]$, shown in Fig. A.1; the resulting amplification of relative errors is illustrated in Fig. A.2, and

Z

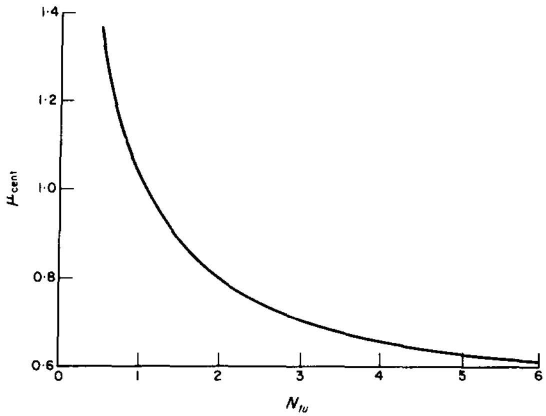

FIG. A.1. Centroid coordinates (fixed clip-off at μ 2·5).

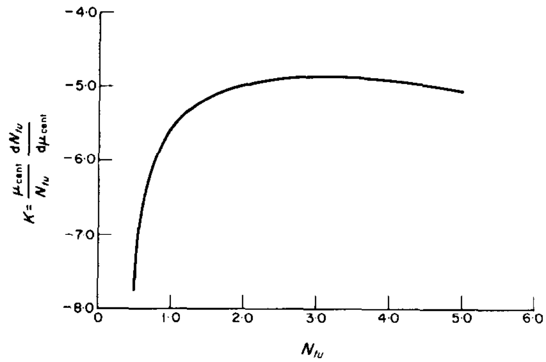

FIG. A.2. Amplification of relative errors (fixed clip-off at $\mu = 2·5$).

seen to be worse than that resulting from the response dependent clip-off shown in Fig. 4. Apart from this shortcoming, no empirical formula corresponding to equation (43) could be found for the centroid coordinate given by equation (68), thus rendering the fixed clip-off approach unsuitable as an off-computer method.

Résumé—On décrit une méthode qui permet de convertir les résultats expérimentaux de transport de chaleur transitoire de matrice dans la gamme des faibles N_{tu} (0,5 < N_{tu} < 5,0). Cette méthode (methode du barycentre) est basée sur (1) la solution analytique du problème du soufflage unique pour des variations arbitraires (décroissantes d'une façon monotone) de la température du fluide amont et sur (2) la jonction indirecte des courbes de réponse de la température du fluide aval par une fonctionnelle univoque qui décroît d'une façon monotone avec N_{tu}, à l'aide spécifiquement de la coordonnée du barycentre de la surface en-dessous de la différence de ces deux températures. Les valeurs de la coordonnée du barycentre sont présentées dans le cas où le changement de la température amont est une fonction-échelon (refroidissement brusque). Ces résultats sont complétés par des facteurs d'amplification d'erreurs provenant d'une analyse linéaire de ces erreurs. Pour le cas plus général de changements arbitraires de la température du fluide amont, deux processus différents sont suggérés pour la conversion des résultats expérimentaux du transport de chaleur transitoire de matrice: soit l'emploi d'un calculateur qui est programmé pour exécuter la méthode du barycentre, ou l'emploi d'une formule empirique qui donne d'une façon approchée la coordonnée du barycentre.

Zusammenfassung—Es wird ein Verfahren beschrieben, das im niedrigen N_{tu}-Bereich (0,5 < N_{tu} < 5,0) für verschiedene Speichermassen die Auswertung der von nichtstationären Wärmeübertragungsversuchen herrührenden Messergebnisse ermöglicht. Dieses Verfahren (Schwerpunktsverfahren) beruht auf (1) der analytischen Lösung für den zeitlichen Temperaturverlauf in der Speichermasse für beliebige (monoton abnehmende) Eintrittstemperaturen des durchströmenden Gases und (2) einem indirekten Kurvenvergleich der Gastemperaturen am Austrittsquerschnitt mit Hilfe eines eindeutigen Funktionals, das mit N_{tu} monoton abnimmt, nämlich die Schwerpunktskoordinate der von der Temperaturdifferenz abgegrenzten Fläche. Werte der Schwerpunktskoordinaten werden für den Fall einer durch eine Stufenfunktion bestimmten Eintrittstemperatur (plötzliches Abkühlen) angegeben. Diese Werte werden durch die, von einer linearen Fehlerabschätzung gelieferten Fehlerfortpflanzungsfaktoren ergänzt. Für den allgemeinen Fall einer beliebigen zeitlichen Änderung der Gastemperatur am Eintrittsquerschnitt werden zwei verschiedene Methoden für die Auswertung von Messergebnissen vorgeschlagen: entweder die Verwendung eines im Sinne des Schwerpunktsverfahrens programmgesteuerten Rechenautomaten oder der Gebrauch einer empirischen Näherungsformel für die Schwerpunktskoordinaten.

Аннотация—Приводится метод обработки экспериментальных данных по нестационарному теплообмену при низких числах N_{tu} (метод центроида). Он основан на: (1) аналитическом решении задачи одночного возмущения для монотонно убывающей температуры вверх по потоку и на (2) косвенном согласовании температурных возмущений с помощью однозначного оператора, монотонно убывающего с N_{tu}, в частности, с помощью центроидной координаты участка, на котором действует разность этих двух температур. Приведены значения центроидной координаты для случая, когда изменение температуры вверх по потоку является ступенчатой функцией (внезапное охлаждение). Полученные данные дополняются коэффициентами увеличения погрешности, полученными в результате линейного анализа погрешности. Для более общего случая произвольных изменений температуры вверх по течению предложены два других метода обработки матричных данных нестационарного теплообмена: Использование вычислительной машины, запрограммированной для центроидного метода, или применение эмпирической формулы, аппроксимирующей центроидную координату.